Derivatives - Vol. 2

Derivatives of Trigonometric functions

Derivative of Sine function

Since $\sin(x+h) = \sin x \cdot \cosh + \sinh \cdot \cos x$

if $f(x) = \sin x$, then

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin x \cdot \cosh + \sinh \cdot \cos x - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x(\cosh - 1) + \sinh \cdot \cos x}{h}$$

$$= \lim_{h\to 0} \sin x \left(\frac{\cosh - 1}{h}\right) + \lim_{h\to 0} \left(\cos x \cdot \frac{\sinh}{h}\right)$$

$$= (\sin x \cdot 0) + (\cos x \cdot 1) = 0 + \cos x = \cos x$$

$$\boxed{\text{so, } \sin' x = \frac{d}{dx} \sin x = \cos x}$$

Derivative of Cosine function

Since $\cos(x+h) = \cos x \cosh - \sin x \sinh$,

if $f(x) = \cos x$, then,

$$\frac{d}{dx} f(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\cos(x+h) - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x \cosh - \sin x \sinh - \cos x}{h}$$

$$= \lim_{h\to 0} \frac{\cos x(\cosh - 1) - \sin x \sinh}{h}$$

$$= \lim_{h\to 0} \cos x \left(\frac{\cosh - 1}{h}\right) - \lim_{h\to 0} \sin x \left(\frac{\sinh}{h}\right) =$$

$\cos x \cdot 0 - \sin x \cdot 1 = -\sin x$. so, $\boxed{\frac{d}{dx} \cos x = -\sin x}$